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# THE MTI IMPROVEMENT FACTOR BY USING DOUBLE - DELAY FILTER AT TWO AND THREE PERIOD STAGGERED

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***ABSTRACT***

***Ground clutter affects the ground based radar. The higher the radar is above the ground. The greater will be the amount of clutter the radar will "see" in the ground - based radar, clutter signals are primarily from a fixed and permanent target buildings, towers and other manmade structures give move intense echo signals than ordinary country side because of the presence of flat reflecting surfaces and corner reflectors.***

***Many of the recent sophisticated radar systems are characterized by the fact that they have to acquire and track small targets of large distances and at angles close to the horizon. In order to meet these requirement, high radiated power levels and increased receiving system sensitivity are frequently required. As a result, several problems arise which can severely limit radar performance. While these problems are different in effect. They are similar in their solution requires to control over the radar site environment.***

***KEYWORDS: Two and Three Period Staggered, Radar, MTI System & Spectrum***

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**Original Article**

## 1. INTRODUCTION

Many of the recent sophisticated radar systems are characterized by the fact that they have to acquire and track small targets at large distances and at angles close to the horizon.

In order to meet these requirements, high radiated power levels and increased receiving system sensitivity are frequently required.

As a result, several problems arise which can severely limit radar performance while these problems are different in effect.

They are similar in their solution require over the radar site environment, the first problem of concern is the clutter return to the radar caused by the reflection of the transmitted signal from mountains or other terrain surrounding the radar sites in practice, it has often been found infeasible to discriminate sufficiently against this clutter filtering or MTI techniques.

## 2. QUANTITATIVE ANALYSIS OF THE MTI SYSTEM

### 2.1. Definition of the MTI System

One respect in which clutter masses differ from actual targets in that they move at different relative speeds.

In MTI system, the velocity characteristics conveyed by the Doppler shift are used to distinguish the desired from undesired returns. This is accomplished in the principle by employing filter networks (or their equivalent), which annul the returns from the fixed targets bearing to Doppler shift while passing the frequency- shifted echoes from moving targets. It is therefore, apparent that clutter elimination by these means is feasible to the extent that the energy spectrum returned by the clutter mass is distinguishable from that of the target. Actually, this is seldom the case. Either because of random fluctuations in scattering cross section, or because of random notions within the clutter mass, the power spectrometers a continuous range of frequencies, and those components in the vicinity of the target frequencies are indistinguishable from target energy. Thus, depending on the width of the clutter spectrum in relation to the range of target velocities to be accepted, a residue of inconsolable clutter persists. (1)

Various other factors contribute to the residual clutter. Briefly, there are "wind noise" caused by unified motion of the clutter mass; "scanning noise" due to modulation of the clutter return by the scanning antenna; "plateform- notion noise", occurring in airborne or shipboard radars and caused by translation of the antenna, and "instability noise", which is due to imperfection in the radar system.(2)

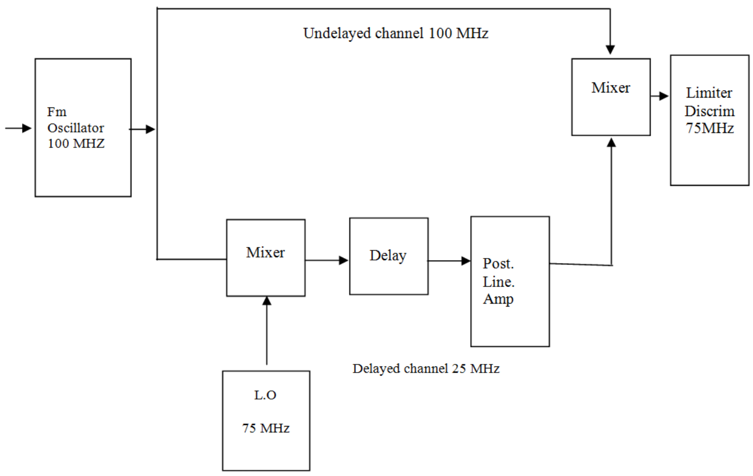


**Figure 2.1: Block Diagram of FM Canceller**

For the purpose of derivation we shall use a simple block diagram for a single loop canceller as shown in the Figure 2.2

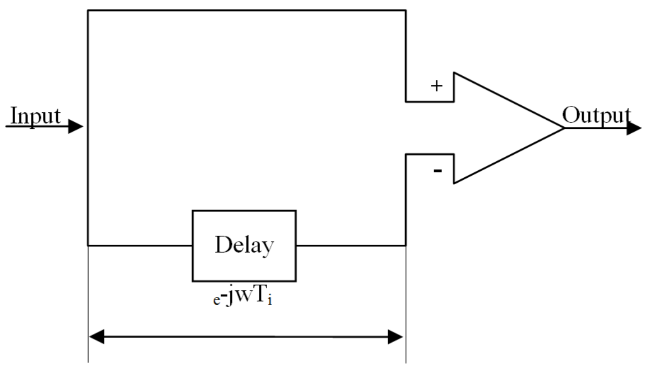


**Figure 2.2: Single Loop Canceller**

Where:

$e^{-jwTi}$ = the frequency response of the delay line

wTi: the phase shift in time domain

Ti: pulse period

**2.2. MTI Filter Characteristics**

The delay line canceller acts as a filter which rejects the dc component of the clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and it is harmonics.

The MTI canceller is in fact a filter possessing a periodic transmission characteristics having zeroes which are corresponding to individual lines of the spectrum of the radar pulse as shown in the Figure 2.3

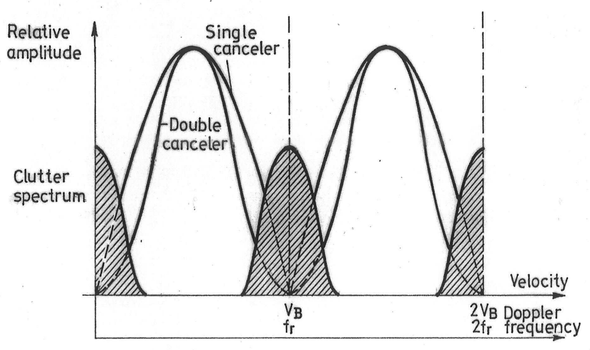


**Figure 2.3: MTI Velocity Responses**

and the black diagram of the FM canceller denoted in the Figure 2.3

$$H(jw) = 1 - e^{-jwTi} \qquad (2.1)$$

By taking the magnitude of this expression

$$|H(jw)| = (1 - \cos wTi)^2 + (\sin wTi)^2 \qquad (2.2)$$

Therefore,

$$|H(jw)| = 2\sin\frac{\Pi ft}{fr} \text{ in voltage response} \qquad (2.3)$$

Where:

$f_t$ = transmitted frequency in MHz

$f_r$ = pulse repetition frequency in Hzc

It has been seen that if the ratio $f_t$ / $f_r$ = is the integer number, the frequency response characteristics are equal to zero, that is, mean a blind speed occurred in the transfer function. But if $f_t$ / $f_r$ = 1, that means the transfer function characteristic is maximum, as shown in the fig. 2.4

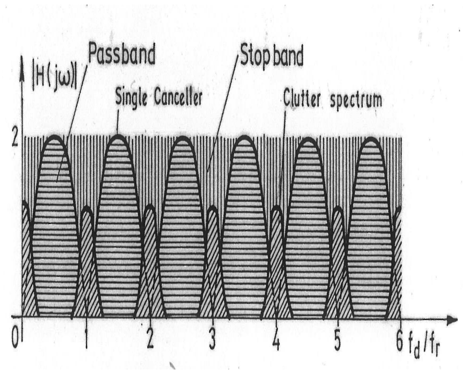


**Figure 2.4: Transfer Function Curve with Clutter Spectrum**

The sharp rejection notch provided by the single- delay canceller is not adequate in cases where the clutter signal spreads beyond in about one percent of the radar repetition.

## 3. IMPROVEMENT FACTOR

The video spectrum may now be applied to the radar filter. Its function is to reduce the clutter or noise while preserving the target signal, which it does by attenuating those parts of the spectrum. More likely to be occupied by interference than by target. After filtering, the Output Ratio (ro) of target -to- clutter or signal-to-noise is computed and compared to the ratio (ri) found in the input to the receiver. The ratio ro/ ri is the improvement factor (I) which is the figure of merit of the filter. (2)

The calculation of clutter residue implies the evaluations of the improvement factor, which is the parameter that describes the behaviour of the canceling circuit. This parameter, defined as the average improvement in the signal-to-clutter ratio introduced by the canceller, describes the cancellation including the canceller average gain.

$$I = \frac{S_o / C_o}{S_i / C_i} \tag{3.1}$$

Where the signal is intended to be averaged over echoes from moving targets, considering all possible radial speeds equiprobable.

The improvement factor is also related to the more widely used clutter attenuation by the noise gain $\overline{G}$ between the output and the input of the canceller.

$$I = \overline{G}.CA \tag{3.2}$$

The analytical expression of the frequency transfer function given as:

$$|H_1(f)| = 2\sin(\frac{\pi fd}{fr}) \tag{3.3}$$

$$|H_2(f)| = 4\sin^2(\frac{\pi fd}{fr}) \tag{3.4}$$

For the single and double canceller, respectively,

In the time domain, the canceller effect may be easily investigated through the auto correlation function in the input to the canceller, $R_o(t)$ the single and double canceller perform respectively, a comparison between two and three successive pulses, spaced by pulse repetition period T.

Therefore, the improvement factor in the two cases is expressed by:

$$I_i = \frac{R_o(0)}{R_o(0) - R_o(T)} \tag{3.5}$$

$$I_2 = \frac{R_o(0)}{3R_o(0) - 4R_o(T) + R_o(2T)} \tag{3.6}$$

From the above expression, it is obvious that the presence of nonlinearities before the canceller, which after the auto correlation, has an influence on the improvement factor. It is therefore necessary to evaluate now $R_o(T)$ in the input of the canceller is bound to the input auto correlation $R_i(t)$.

These two expressions are completely general and allow the evaluation of the improvement factor for any form of the input auto correlation. In order to carry out the numerical computation of the influence of the limitation on MTI performance, the case of a Gaussian input auto correlation function has been examined.

This choice is justified by the fact that the clutter spectrum due to internal fluctuation is always approximated with a Gaussian shape. Furthermore, one of the main causes of clutter spreading antenna motion may be considered to give rise to a Gaussian spectrum.(7).

Since the Gaussian form may closely approximate the usual antenna beam shape.

For the concentrated clutter the approximation of the antenna pattern by a Gaussian shape cannot be accepted with a strong limitation, since the side lobes acquire an increasing importance. Considering instead a distributed clutter, since the whole antenna diagram is involved at any time. Therefore, the contribution from all clutter elements will be reduced, more or less proportionally by the limiter.

Also taking into consideration radar internal instabilities, the best choice of clutter spectrum shape is still the Gaussian. Another advantage offered by a Gaussian spectrum is that the corresponding and auto correlation function is still Gaussian and its powers are still Gaussian. (6)

If we return to the essential definitions of the improvement factor (I) as denoted in the eq. (3.1) we shall take the single loop canceller to derive the expression of improvement factor.

We have:

G: the gain of the MTI filter = $S_o$ / $S_i$

$C_i$ / $C_o$: which expresses the MTI clutter attenuation

(i.e. The ratio of clutter echoes at the canceller input to clutter residue at the output).

By finding the area under the curve in the figure (4.14) that means the gain of the MTI filter.

$$G = \int_0^{\infty} 4\sin^2(\pi \frac{f}{fr})df = 2 \tag{3.7}$$

Which is equivalent to the average

The power gain of the MTI filter, from

Gaussian shape we have the spectrum

power at the input filter is

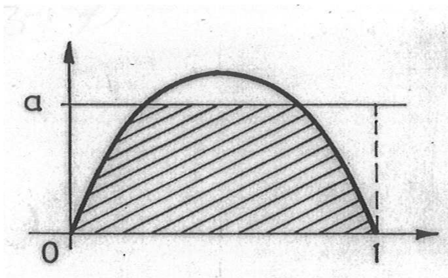


**Figure 3.1: Frequency Response Shape for Single Loop Canceller**

$$G_i = \int_{-\infty}^{\infty} k \ e^{-} \frac{f^2}{2\sigma^2} df \tag{3.8}$$

Where σ is the standard deviation of the spectrum clutter. By solving this integral we shall get

$$C_i = \sqrt{2\pi} k\sigma \tag{3.9}$$

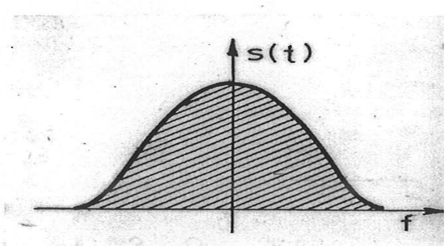


**Figure 3.2: Gaussian Shape**

The output spectrum power from the filter is

$$C_0 = \int_{-\infty}^{\infty} k \, e^{\frac{-f^2}{2\sigma^2}} \quad |H(jw)|^2 df \tag{3.10}$$

Where |H(jw)| is the frequency response of the filter. By solving this integral we shall get:

$$C_0 = 2 \quad \sqrt{2\pi} K\sigma \qquad (1 - e^{-2\pi^2\sigma^2 Ti^2})$$

Therefore:

The improvement factor is:

$$I = \bar{C} \qquad C_i / C_0$$

i.e

$$I = 2\frac{\sqrt{2\pi}k\ \sigma}{2\sqrt{2\pi}k\sigma(1-e^{-2\pi^2\sigma^2Ti^2})}$$

$$I = 2\frac{1}{1-e^{-2\pi^2\sigma^2Ti^2}} \tag{3.11}$$

This equation which represents the improvement factor for the single canceller.

**3.1. Blind Speed Problem**

The interval between radar pulses may be changed to shift the target velocities to which the MTI system is blind, i.e. the non-zero blind speeds for the MTI are dependent on the interpulse spacing; a target moving one half wave length between pulses will be lost from view. The interval may be changed on either a scan-to-scan or pulse-to-pulse. Each approach has advantages. The advantages of the scan-to-scan method are that the radar system is easier to blind, and multiple- time around clutter is cancelled in a power- amplifier MTI system.

For many applications, however, pulse-to-pulse staggering is essential. For example, if a no- feedback dual canceller, which has a 36 percent-wide rejection notches is employed, and scan-to-scan pulse staggered is used, 36- percent of the desired target, would be missing an each scan owing to Doppler consideration alone.

With pulse- to- pulse staggering, good response can be obtained on all dopplers of interest on each scan. In addition, better velocity can be obtained at some of the dopplers than either pulse interval will give on a scan- to- scan basis. This is because pulse- to- pulse staggering modulates the Doppler at a frequency close to the maximum response frequency of the canceller.

It has been concluded that the blind speeds are target speeds for which the Doppler $f_d = k\frac{2v}{\lambda}$ falls in a stop band of the MTI filter.

$$f_d = k\frac{2v}{\lambda} \tag{3.12}$$

It can be created another expression which denote the number of blind speeds.

## 4. MTI IMPROVEMENT FACTOR AND THE BLIND SPEEDS "PRACTICAL CALCULATION AND OPTIMIZATION"

### 4.1. Double-Delay MTI Filter with non- Staggered Case

It was shown in item three when it was derived the expression of the improvement factor for single delay MTI filter

$$I = \frac{1}{1-e^{-2\pi^2T^2\sigma^2}} \tag{4.1}$$

Where:

σ: the standard deviation of the clutter

T: the time period

In this item we are going to derive the expressions of the improvement factor for double delay MTI, non-staggered case and staggered case, for two period and three period staggered respectively, and to optimize the useful deepest minimum situation for a frequency response characteristics.

Now, let us assume a non-staggered case first.

The power transfer function for double delay MTI filter is

$$\left|H_2(w)\right| = \left|H_1(w)\right|^2 .$$

Where:

$$\left|H_1(w)\right| = 4\sin_2 \pi \frac{f}{fr} \tag{4.2}$$

The average gain of the MTI filter $\overline{G}$ is:

$$\overline{G} = |\overline{H(f)}| = \frac{1}{f_1}\int_0^{f_1} |H(f)|^2\, df \tag{4.3}$$

When

$$\overline{G} = \frac{1}{f_1}\int_0^{f_1} 16\sin^4 \pi \frac{f}{fr}\, df \tag{4.4}$$

Where:

T= 1/ fr pulse repetition period.

By simplifying the equation we get:

$$\overline{G} = \frac{16}{f_1}\int_0^{f_1} \frac{1}{8}(\cos 4af - 4\cos 2af + 3)\, df \tag{4.5}$$

Where:

a= π T

The depths of the minimum in $|H(f)|_2$ are analyzed through the purposing and plotting of $|H(f)|^2$ in the region from

F = 0 to f = N/T

Because the first zero of the $|Hs(t)|^2$, is evidently when

$$F = N/T \tag{4.6}$$

This assumption shall be clearly seen as going on forward.

By solving the integration and substituting the limits $(0-\frac{N}{T})$ with regard of (N) integer number is obtained:

$\overline{G} = 3$

Now to obtain the improvement factor of the double- delay MTI in case of non- staggering, we have to follow the basic definition of the improvement factor which is:

$$I = \overline{G}\,\frac{c_i}{c_0} \tag{4.7}$$

Where $c_i$ is the clutter line in the input of MTI filter, and $c_0$ is the clutter residue at the output of MTI filter,

$$C_i = \int_{-\infty}^{\infty} k,\, e^{\frac{-f^2}{2\sigma^2}}\, df \tag{4.8}$$

and

$$C_0 = \int_{-\infty}^{\infty} k.\quad e^{\frac{-f^2}{2\sigma^2}} \;|H(f)|^2\, df \tag{4.9}$$

Then:

$$I = 3\frac{\int_{-\infty}^{\infty} k.\; e^{\frac{-f2}{2\sigma^2}}\, df}{\int_{-\infty}^{\infty} k.\; e^{\frac{-f2}{2\sigma^2}} \;|H(f)|^2\, df} \tag{4.10}$$

Solving the above expression with the aid of integration tables to obtain the improvement factor, then:

$$I = 3.\frac{\frac{1}{2}\sqrt{2\pi\sigma}}{16.\frac{1}{8}\int_{-\infty}^{\infty}(\cos 4af - 4\cos 2af + 3) - e^{\frac{f^2}{2\sigma^2}}\, df}$$

Then:

$$I = 3.\frac{\frac{1}{2}\sqrt{2\pi\sigma}}{\frac{1}{2}\sqrt{2\pi\sigma}e\frac{-16\pi^2T^2\sigma^2}{4}\tau^2 - 4\frac{1}{2}\sqrt{2\pi\sigma}e\frac{-4\pi^2T^2}{4}2\sigma^2 + \frac{1}{2}\sqrt{2\pi\sigma}3}$$

Finally,

$$I = \frac{1}{1+\frac{1}{3}e^{-8\pi^2T^2\sigma^2} - \frac{4}{3}e^{-2\pi^2T^2\sigma^2}} \tag{4.11}$$

A computer program is done for the expression (4.11) and the results are plotted in (4.1) which indicate the improvement factor for double delay MTI filter for nonstaggered case in (dB), vrs. The standard deviation in (Hz).

**4.2. Double- Delay MTI Filter with Two- Period Staggered**

To express the formula of a frequency response for double delay MTI, two periods staggered. By using the simple diagram figure 4.1, diagrams: can be stated that:

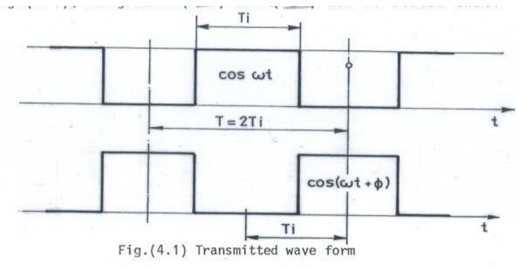


Figure 4.1**: Transmitted Wave Form**

$$S_1(t) = \frac{a_0}{2} + \sum a_k \cos k\frac{\pi}{T_1}t \tag{4.12}$$

Similarly,

$$S_2(t) = S_i(t - T_i) = \frac{a_0}{2} + \sum a_k \cos k\frac{\pi}{T_i}(t - T_i) = \frac{a_0}{2} + \sum a_k \cos(k\frac{w_i}{2}t - k_\pi) \tag{4.13}$$

For k integer

$$\cos k\pi = (-1)^k$$

Therefore:

$$S_2(t) = \frac{a_0}{2} + \sum (-1)^k ak \cos k\frac{w_i}{2}t$$

From fourier series we have:

$$a_k = \frac{1}{T_i}\int_{-T_i/2}^{T_i/2} \cos k \frac{\pi}{T_i} t = \frac{1}{T_i}\frac{T_i}{k} \sin k\frac{\pi}{T_i} \Big|_{-T_i/2}^{T_i/2}$$

Therefore:

$$a_k = \frac{2}{k_\pi}\sin\frac{k_\pi}{2} \tag{4.14}$$

Then

The value of the odd number is

K = 2 n – 1 $\quad a_k = \pm 1$

The value of the even number is:

K = 2 n $\qquad a_k = 0$

Therefore (k) is an odd number:

Then

$(-1)^k = -1$

So,

$$S_2(t) = \frac{a_0}{2} - \Sigma a_k \cos k\frac{\pi}{T_i}t = \frac{a_0}{2} - \Sigma a_k \cos k\frac{w_i}{2}t$$

Then

$f_1(t) = S_1(t)\cos wt$

$$f_1(t) = \frac{1}{2}\cos wt + \frac{1}{2}\Sigma a_k \cos(w + k\frac{w_i}{2}) + \frac{1}{2}\Sigma a_k \cos(w - \frac{w_i}{2})t \qquad (4.15)$$

Similarly

$$f_2(t) = S_2(t)\cos(wt - w\Delta T)$$

Then

$$f_2(t) = \frac{1}{2}\cos(wt - w\Delta T) - \frac{1}{2}\Sigma a_k \cos|(w + k\frac{w_i}{2})t - w\Delta t| - \frac{1}{2}\Sigma a_k \cos|(w - k\frac{w_i}{2})t - w\Delta t|$$

(4.16)

Therefore:

$$f(t) = f_1(t) + f_2(t) \qquad (4.17)$$

$$f(t) = \cos(wt - w\frac{\Delta T}{2})\cos w\frac{\Delta T}{2} - \Sigma a_k \sin|(w + k\frac{w_i}{2})t - w\frac{\Delta t}{2}|\sin w\frac{\Delta t}{2} - \Sigma a_k \sin|(w - k\frac{w_i}{2})t - \frac{w\Delta t}{2}|\sin w\frac{\Delta t}{2}$$

Then:

$$f(t) = \cos w\frac{\Delta T}{2}\cos(wt - w\frac{\Delta T}{2}) - \sin w\frac{\Delta T}{2}\Sigma a_k \sin|(w + k\frac{w_i}{2})t - \frac{w\Delta t}{2}| - \sin w\frac{\Delta t}{2})\Sigma a_k \sin|(w - \frac{w_i}{2})t - \frac{w\Delta t}{2}|$$

The pulse envelope at the input to the canceller has been resolved into three components:

- A sinusoidal envelope of component (w), amplitude cos $w\frac{\Delta T}{2}$.
- A sinusoidal envelope of component $(w + k\frac{w_i}{2})$ amplitude $\sin w\frac{\Delta T}{2}$.

- A similar envelope of component $(w - k\frac{w_i}{2})$

Finally,

$$f_{0p}-(t)=H(w)\cos w\frac{\Delta T}{2}\cos(wt-w\frac{\Delta T}{2})+\sin w\frac{\Delta T}{2}\Sigma a_k H(w+k\frac{w_i}{2})\sin|(w+k\frac{w_i}{2})t-\frac{w\Delta t}{2}|+\sin\frac{w\Delta T}{2}\Sigma a_k H(w-k\frac{w_i}{2})\sin|(w-\frac{w_i}{2})t-\frac{w\Delta t}{2}|$$

The frequency response of the canceller is periodic in ($w_i$) so that $H(w+k\frac{w_i}{2}) = H(w-k\frac{w_i}{2})$, and the two "sideband" responses can be added together.

Therefore:

$$f_{0|p}(t) = H(w)\cos w\frac{\Delta T}{2}\cos(wt - w\frac{\Delta T}{2}) - H(w+k\frac{w_i}{2}).\sin w\frac{\Delta T}{2}\Sigma a_k\{\sin\ (w+k\frac{w_i}{2})t - \frac{w\Delta t}{2}\} + \sin\,(w-k\frac{w_i}{2})t - \frac{w\Delta t}{2}\}\,|$$

Simplifying the above expression to get:

$$f_{0|p}(t) = H(w)\cos w\frac{\Delta T}{2}\cos(wt - w\frac{\Delta T}{2}) - 2H(w+\frac{w_i}{2})\sin w\frac{\Delta T}{2}.\sin(wt-\frac{w\Delta t}{2})\Sigma a_k\cos\frac{kw_i}{2}t \quad (4.18)$$

From fourier series we have:

$$S_1(t) = \frac{a_0}{2} + a_k\cos k\frac{w_i}{2}t$$

Then

$$\Sigma a_k\cos k\frac{w_i}{2}t = S_1(t) - \frac{a_0}{2}$$

We have $a_0 = 1$

Then

$$a_k\cos k\frac{w_i}{2}t = S_1(t) - \frac{1}{2}$$

Because it was assumed a sinusoidal waveform as shown in Figure 4.2, then:

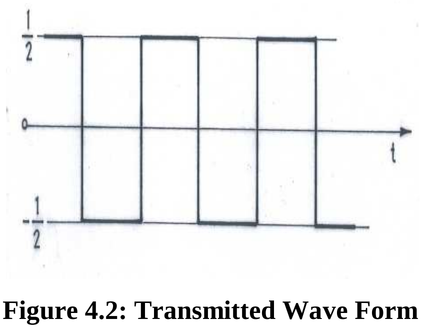


**Figure 4.2: Transmitted Wave Form**

$$\Sigma a_k \cos k\frac{w_i}{2}t = \pm\frac{1}{2} \tag{4.19}$$

$$f_{0|p}(t) = H(w)\cos w\frac{\Delta T}{2}\cos(wt - w\frac{\Delta T}{2}) - 2H(w+\frac{w_i}{2})(\pm\frac{1}{2})\sin w\frac{\Delta T}{2}.\sin(wt - w\frac{\Delta t}{2})$$

Now, let us assume,

$$H(w)\cos w\frac{\Delta T}{2} = A$$

$$wt - w\frac{\Delta T}{2} = \alpha$$

$$2H(w+\frac{w_i}{2})(\pm 1/2)\sin w\frac{\Delta T}{2} = B$$

Then:

$$f_{0|p}(t) = A\cos\alpha \pm B\sin\alpha$$

$$\frac{1}{(f_{0|p}(t))^2} = \frac{1}{(A\cos\alpha \pm B\sin\alpha)^2} = \frac{1}{(A^2\cos^2\alpha \pm 2AB\sin\alpha\cos\alpha + B\sin^2\alpha)}$$

But,

$$\bar{f}_{0/p} = \frac{1}{2\pi}\int_0^{2\pi} f_{0|p}^2\,dt$$

Then

$$A^2\frac{1}{2\pi}\int_0^{2\pi}\cos^2\alpha\; dt = \frac{A^2}{2}$$

$$\pm 2AB\frac{1}{2\pi}\int_0^{2\pi}\cos\alpha\sin\;\alpha dt = 0$$

$$B^2\frac{1}{2\pi}\int_0^{2\pi}\sin^2\alpha\; dt = \frac{B^2}{2}$$

Then:

$$f^2{}_{0/p} = \frac{1}{2}A^2 + \frac{1}{2}B^2$$

$$= \frac{1}{2}|\,H^2(w)\cos^2 w\frac{\Delta T}{2} + H^2(w+\frac{w_i}{2})\sin^2\frac{w\Delta T}{2}\,|$$

Finally,

$$|\overline{H_s(w)}|^2 = \frac{\overline{f_{out}}}{f_{in}^2} = |H(w)|^2 \cos^2\frac{w\Delta T}{2} + |H(w+\frac{w_i}{2})|^2 \sin^2\frac{w\Delta T}{2} \tag{4.20}$$

$$|\overline{H_s(w)}|^2 = \sin^4(\pi fT)\cos^2(\pi f\Delta T) + \cos^4(\pi fT)\sin^2(\pi f\Delta T) \tag{4.21}$$

When:

$T_1 = T + \Delta T$

$T_2 = T - \Delta T$

T: mean period and its is equal to the delay in canceller.

$\Delta T$: stagger delay

Then:

$T = \frac{1}{f_r}$, $f_r$ mean repetition frequency

Putting:

$$\frac{\Delta T}{T} = \frac{M}{N}$$

Therefore:

$$\Delta T = T\frac{M}{N} \tag{4.22}$$

The power transfer characteristic will be:

$$|\overline{H_s(t)}|^2 = \sin^4(\pi fT)\cos^2(\pi fT\frac{M}{N}) + \cos^4(\pi fT)\sin^2(\pi fT\frac{M}{N}) \tag{4.23}$$

Let us assume that:

$V_b$: first blind speed for non-staggered case.

$V_{b'}$ : first blind speed for staggered case.

We have that:

$$V_b = \frac{\lambda}{2} \quad . \quad f_r = \frac{\lambda}{2T} \tag{4.24}$$

To obtain $V_{b'}$ we must look for the first zero of the $|H_s(f)|^2$ , evidently the zero is when

$$f = \frac{N}{T}$$

Putting

$$f = \frac{2V_{b'}}{\lambda}$$

But,

$$f = \frac{N}{T} \tag{4.25}$$

Then,

$$\frac{N}{T} = \frac{2V_{b'}}{\lambda}$$

$$V_{b'} = N\frac{\lambda}{2T} = NV_b \tag{4.26}$$

So, $|H_s(f)|^2$ can be written in another form as follows:

$$f = \frac{2v}{\lambda} \qquad\qquad T = \frac{\lambda}{2V_b}$$

So, $\pi fT = \pi(\frac{v}{v_b})$

From equation (4.25)

$$v_{b'} = Nv_b$$

Then

$$\pi fT = \pi N(\frac{v}{v_{b'}})$$

Finally,

$$\pi fT\frac{M}{N} = \pi M(\frac{v}{v_{b'}}) \tag{4.27}$$

So, the frequency response characteristic according to the last form shall be:

$$|H_s(v)|^2 = \sin^4(\pi N\frac{v}{v_{b'}})\cos^2(\pi M\frac{v}{v_{b'}}) + \cos^4(\pi N\frac{v}{v_{b'}})\sin^2(\pi M\frac{v}{v_{b'}}) \tag{4.28}$$

**4.3. The Mean Period of the Main Delay Line N= 30**

The deepest minimum has occurred when the staggered delay (M) is an even number with a value of (-50dB), the worst case obtained when the staggered ratio is 1/5 and 2/3, but the best case at M = N-1 Figure 4.2.11.

The deepest minimum for each case, i.e. when the mean period of main delay line (N) are (10, 20 and 30) is plotted vrs. The staggered delay (M) as shown in figures 4.3.1 to 4.3.3 which shows the effect of the odd and the even numbers on the staggering.

We are going now to find the expression for improvement factor in the case of two period stagger by using the double delay MTI filter.

First of all we must find the average gain of the employed MTI filter:

We have:

$$|H_s(f)|^2 = \sin^4(\pi fT)\cos^2(\pi fT\frac{M}{N}) + \cos^4(\pi fT)\sin^2(\pi fT\frac{M}{N}) \tag{4.29}$$

To solving the above equation, assume that:

$$\pi T = a$$

$$\pi T\frac{M}{N} = b$$

$$|H_s(f)|^2 = \sin^4 af\,\cos^2 bf + \cos^4 af\,\sin^2 bf$$

Then

$$|H_s(f)|^2 = 1/8(-4\cos 2af\ \ \cos\ 2bf + \cos 4af + 3) \tag{4.30}$$

From the definition of frequency response characteristic

$$|H_s(f)|^2 = 16.1/8(-4\cos 2af\cos 2bf + \cos 4af + 3) \tag{4.31}$$

The average gain shall be:

$$|\overline{H(f)}|^2 = \frac{1}{f_1}\int_0^{f_1} |H(f)|^2 df \tag{4.32}$$

Then, by the aid of integration tables, obtain

$$|\overline{H(f)}|^2 = \frac{T}{N}\int_0^{N/T}(-4\cos 2af\cos 2bf + \cos 4af + 3)df \tag{4.33}$$

By solving the integral (4.34) we obtain:

$$|\overline{H_s(f)}| = 1/N\left|-\frac{\sin 2\pi N(1+\frac{M}{N})}{\pi(1+\frac{M}{N})} - \frac{\sin 2\pi N(1+\frac{M}{N})}{\pi(1-\frac{M}{N})} + \frac{\sin 4\pi N}{4\pi} + 3N\right|$$

If (M) and (N) are integers:

Result:

$$|H_s(f)|^2 = \frac{1}{N}\left|\frac{-\sin 4\pi N}{2\pi} + \frac{\sin 4\pi N}{4\pi} + 3N\right|$$

Finally:

$$|H_s(f)|^2 = \frac{1}{N} \quad 3N \qquad = 3$$

From the definition of the improvement factor we have:

Improvement factor $(I) = \overline{G}\,\dfrac{C_i}{C_0}$ (4.34)

Then,

$$I = 3.\frac{1/2\sqrt{2\pi\sigma^2}}{\int_0^\infty (-4\cos 2\pi Tf \cos \pi Tf \frac{M}{N} + \cos 4\pi Tf + 3)e^{\frac{-f^2}{2\sigma^2}}\,df}$$

By solving the expression, we got:

$$I = \frac{3}{3 + e^{-8\pi^2T^2\sigma^2} - 2e^{-2\pi^2T^2\sigma^2}(1 + \frac{M^2}{N^2})(e^{-4\pi^2T^2\sigma^2}\frac{M}{N} + e^{-4\pi^2T\sigma^2}\frac{M}{N})}$$

Then:

$$I = \frac{3}{3 + e^{-8\pi^2T^2\sigma^2} - 4e^{-2\pi^2T^2\sigma^2}(1 + \frac{M^2}{N^2}).\cosh(4\pi^2T^2\sigma^2\frac{M}{N})}$$

Finally,

$$I = \frac{1}{1 + \frac{1}{3}e^{-8\pi^2T^2\sigma^2} - \frac{4}{3}e^{-2\pi^2T^2\sigma^2}(1 + \frac{M^2}{N^2}).\cosh(4\pi^2T^2\sigma^2\frac{M}{N})} \tag{4.35}$$

The computer program has been done for the expression (4.35), by taking the improvement factor for two- period-staggered as a function of standard deviation of clutter. The results have been plotted in plots (4.2.9) to (4.2.10).

**4.4. Double- Delay MTI Filter with Three Period Staggered**

By using the same way as in two- period staggered for deriving the expression of frequency response characteristics. Therefore, it has been obtained the expression (4.34) for three period staggered.

$$|H_s(v)|^2 = \sin^4(\pi Tf) - \frac{4}{3}\sin^2(\pi Tf\frac{M}{N})\sin^4(\pi Tf) + \frac{1}{2}\sin^4(\pi Tf\frac{M}{N}) \tag{4.36}$$

A forming of computer program is done for (4.36) and for six cases by choosing the mean period of main delay line (N) as (5, 10, 15, 20, 25 and 30) and changing the staggered delay (M) for each case from (1) to (M= N-1), to be able to analyses and predict the optimum case of staggering.

By employing the expression (3.35), which indicates the frequency response characteristic in case of three- period staggered and assuming that:

$$\pi T = a$$

$$\pi T\frac{M}{N}=b$$

By applying the definition of average gain to get:

$$\overline{G}=\frac{T}{N}\int_{0}^{N/T}(\sin^4 af-\frac{4}{3}\sin^2 bf\sin^4 af+\frac{1}{2}\sin^2 bf)df \tag{4.37}$$

Simplifying this expression to obtain

$$\overline{G}=\frac{T}{N}\int_{0}^{N/T}(1/3\cos 4af-4/3\cos 2af+3+2/3\cos bf\cos 4af-8/3\cos 2bf\cos 2a)df$$

Solving this integral by supposing (M) and (N) are integers, we obtain

G = 3

We are going now to obtain the expression of the improvement factor for three- period stagger.

From the definition of the improvement factor we have.

$$I=G\frac{C_i}{C_0}$$

Where $C_i$ is the clutter at the input of MTI filter and equal

$$C_i=\int_{0}^{\infty}e^{\frac{f^2}{2\delta^2}}\,df$$

and $C_0$ is the clutter residue at the output of MTI filter which equal

$$C_0=\int_{0}^{\infty}|H(f)|^2\,e^{\frac{-f^2}{2\delta^2}}$$

For finding $C_0$, say

$$C_0=\int_{0}^{\infty}(\frac{1}{3}\cos 4\pi Tf.e\frac{-f^2}{2\sigma^2}df-\frac{4}{3}\int_{0}^{\infty}\cos 2\pi Tf.e\frac{-f^2}{2\sigma^2}df+3\int_{0}^{\infty}e\frac{-f^2}{2\sigma^2}df++\frac{2}{3}\int_{0}^{\infty}\cos 2\pi Tf\frac{M}{N}\cos 4\pi Tfe\frac{-f^2}{2\sigma^2}df-\frac{8}{3}\int_{0}^{\infty}\cos 2\pi Tf\frac{M}{N}\cos 2\pi Tf\frac{M}{N}\cos 2\pi Tfe\frac{-f^2}{2\sigma^2}df$$

By the aid of integration tables, this integrals were solved and the results are:

$$C_0=\frac{1}{3}\bullet\frac{1}{2}\sqrt{2\pi}\ \sigma^2\,e^{-8(\pi T\sigma)^2}-\frac{4}{3}\bullet\frac{1}{2}\sqrt{2\pi}\sigma^2\ e^{-2(\pi T\sigma)^2}+3\frac{1}{2}\sqrt{2\pi}\,\sigma^2+\frac{2}{3}\bullet\frac{1}{2}\sqrt{2\pi}\ \sigma^2\ e^{-2(\pi T\sigma)^2}(4+\frac{M^2}{N^2}).$$

$$\cosh 8(\pi^2T^2\sigma^2)\frac{M}{N^-}-\frac{8}{3}\bullet\frac{1}{2}\sqrt{2\pi}\,\sigma^2\,e^{-2(\pi T\sigma)^2}(1+\frac{M^2}{N^2})\cosh 4(\pi T\sigma)^2\frac{M}{N}$$

By applying the definition of improvement factor.

$$I = 3\frac{1/2\sqrt{2\pi}\sigma^2}{\frac{1}{2}\sqrt{2\pi}\sigma^2\left[\frac{1}{3}e^{-8(\pi T\sigma)^2} - \frac{4}{3}e^{-2(\pi T\sigma)^2} + 3 + \frac{2}{3}e^{-2(\pi T\sigma)^2}(4+\frac{M^2}{N^2})\right.}$$

$$\overline{\left.\cosh 8(\pi T\sigma)^2\frac{M}{N} - \frac{8}{3}e^{-2(\pi T\sigma)^2}(1+\frac{M^2}{N^2})\cosh 4(\pi T\sigma)^2\frac{M}{N}\right]}$$

Finally

$$I = \frac{1}{1+\frac{1}{9}e^{-8(\pi T\sigma)^2} - \frac{4}{9}e^{-2(\pi T\sigma)^2} + \frac{2}{9}e^{-(\pi T\tau)^2}(4+\frac{M^2}{N^2})\cosh 8\,(\pi T\tau)\frac{2M}{\bar{N}} - \frac{8}{9}e^{-2(\pi T\sigma)^2}(1+\frac{M^2}{N^2})\cosh 4(\pi T\tau)^2\frac{M}{N}} \tag{4.38}$$

A computer program has been done for (4.38) to indicate the improvement factor for three- period staggered as a function of standard deviation of clutter (σ) in (Hz). The results have drawn in Figures (4.3.5) to (4.3.6).

## RESULTS

The purpose of this research is to investigate and to find the optimum staggered ratio employed in MTI radar for eliminating the blind velocities and enhancing the improvement factor, and to study the effectiveness of active noise on a radar performance.

We found that the optimum case for staggering satisfied the simple relation (M=N-1). In this case the deepest minimum is the first blind speed, the blind speeds are absent near the centre of the response curve, but deep nulls appear at the blind speed near the two ends of the response.

It has been noticed that if the ratio of periods is very close say 29: 30 then the blind speed is increased by a large factor, in this case 30 times. On the other hand, the two components first- order blind speeds are separated by only one part in 30 so that the response falls nearly to zero after the first maximum. The choice of stagger ratio is clearly a compromise between the gap free coverage at high target speeds and deep minim rat low speeds. The final selection had been made by examining computed responses for various ratios.

As the stagger ratio T1/ T2 is increased, the depth of the nulls between the blind speeds in the response characteristics will increase too, and when this ratio is equal (1/3, 1/4, 3/4… etc.) more than one deepest minimum can be obtained in the response curve each equal (- 50dB) and it is considered to be unsatisfactory case. But, the worst case is when the staggered ratio is 1/2 and the last result can be achieved when:

| | | |
|---|---|---|
| a. | N = 10 | M=5 |
| b. | N = 20 | M=10 |
| c. | N = 30 | M=15 |

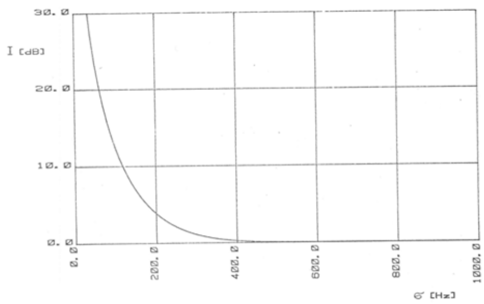


**Figure 4.1.1: Improvement Factor for Non-Staggered Close**

The number of the deepest minimum in the response curve is (4, 9 and 14) this means that the number of the deepest minimum in the response is (M – 1) as shown in the Figures (4.2.1) to (4.2.11) it was also found that if the value of N increases, the value of the improvement factor will be decreased, i.e. we must choose the value of N as small as possible

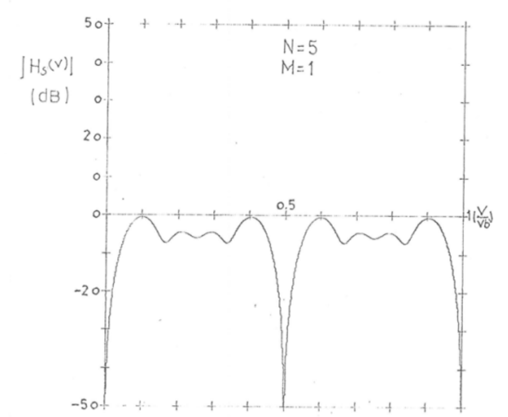


**Figure 4.2.1: Velocity Response of Stagger Ratio 4:6**

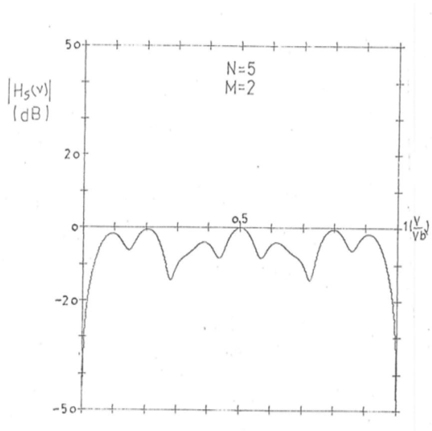


**Figure 4.2.2: Velocity Response of Stagger Ratio 3:7**

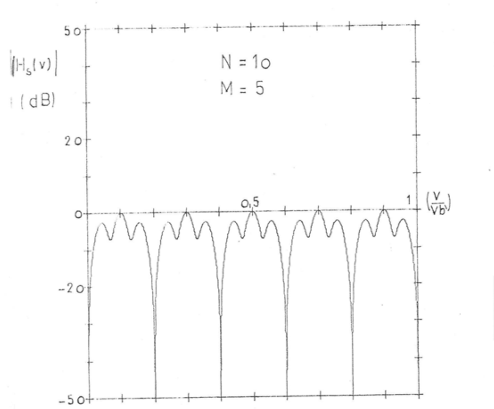


**Figure 4.2.3: Velocity Response of Stagger Ratio 5:15**

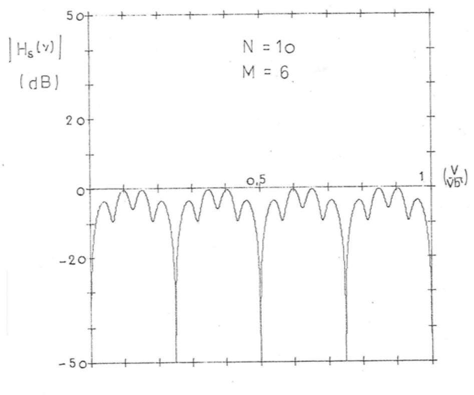


**Figure 4.2.4: Velocity Response of Stagger Ratio 4:16**

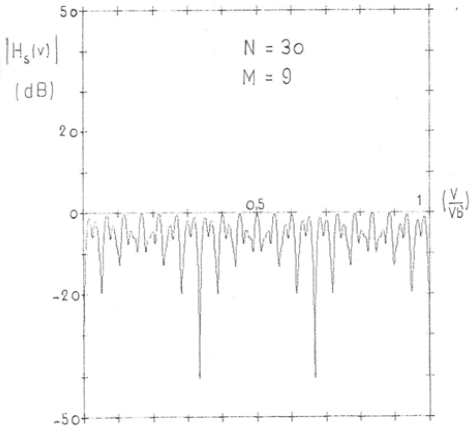


**Figure 4.2.5: Velocity Response of Stagger Ratio 21:39**

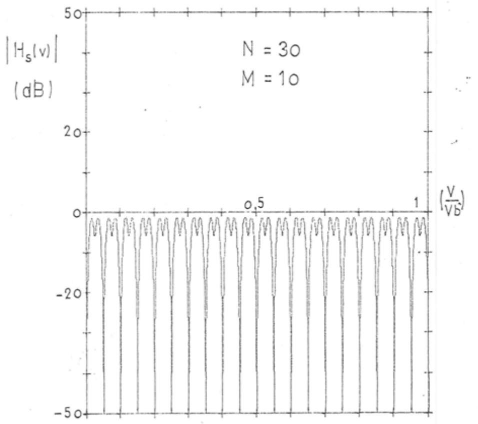


**Figure 4.2.6: Velocity Response of Stagger Ratio 20:40**

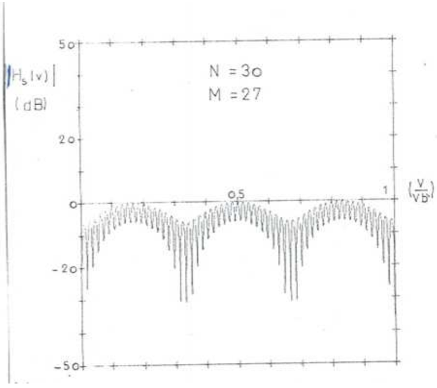


**Figure 4.2.7: Velocity Response of Stagger Ratio 3:57**

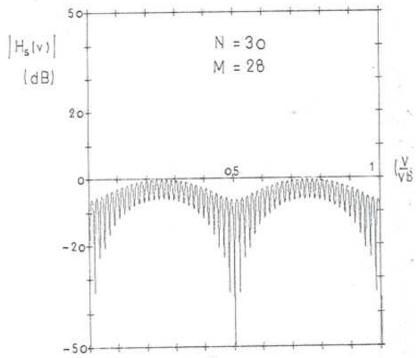


**Figure 4.2.8: Velocity Response of Stagger Ratio 2:58**

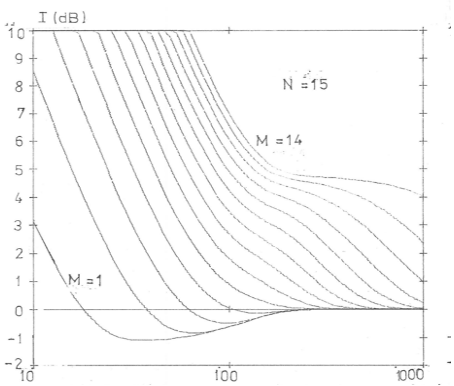


**Figure 4.2.9: Improvement Factor for 5(Hz) Two-Period Stagger**

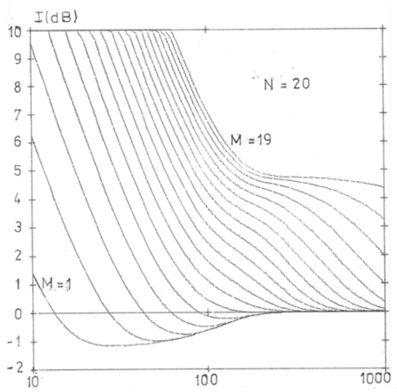


**Figure 4.3.0: Improvement Factor for 5(Hz) Two-Period Stagger**

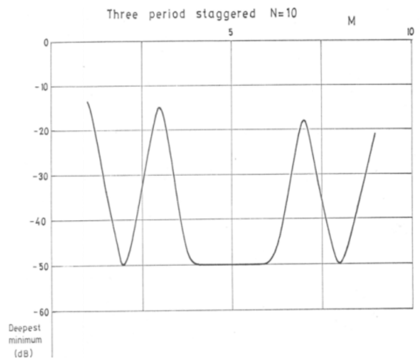


Figure 4.3.1: A Deepest Minimum in Response Curve vrs. Stagger Delay

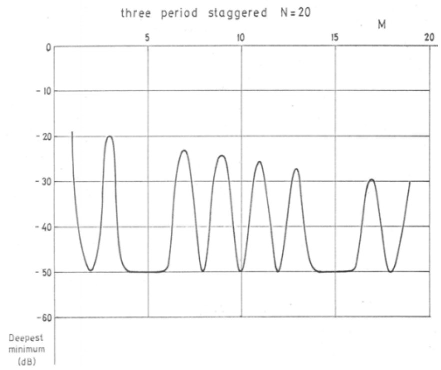


Figure 4.3.2: A Deepest Minimum in Response Curve vrs. Stagger Delay

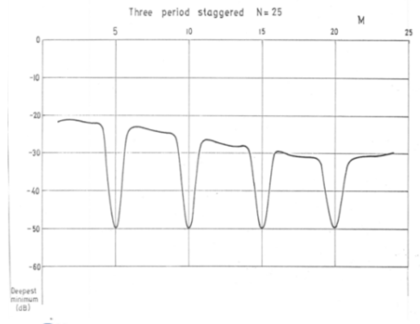


Figure 4.3.3: A Deepest Minimum in Response Curve vrs. Stagger Delay

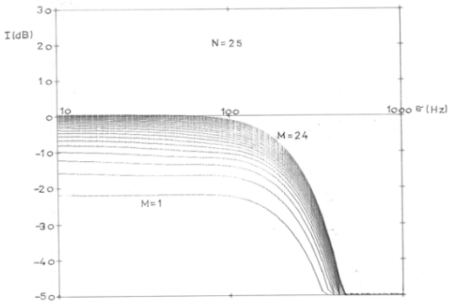


Figure 4.3.4: Improvement Factor for Three Period Stagger

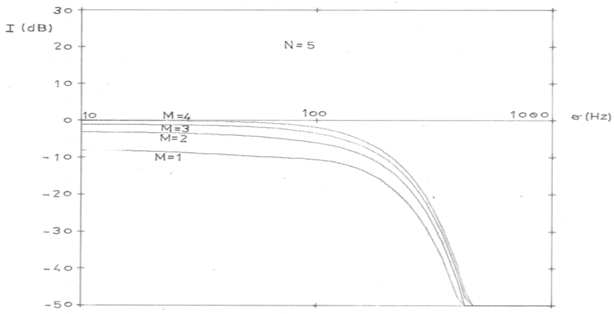


Figure 4.3.5: Improvement Factor for Three Period Stagger

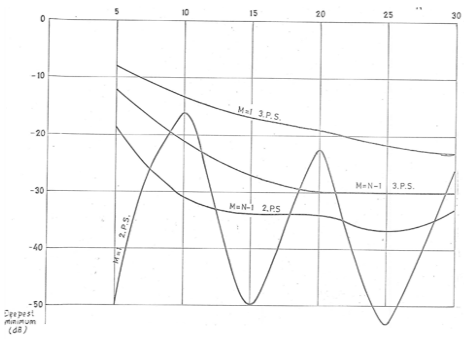


Figure 4.3.6: A Comparison between Two and Three Period Stagger M=1 and M=N-1

## CONCLUSIONS

The purpose of this research is to investigate and to find the optimum staggered ratio employed in MTI radar for eliminating the blind velocities and enhancing the improvement factor.

It has been found that the optimum case for staggering satisfies the simple relation (M=N-I), In this case the deepest minimum is the first blind speed, the blind speeds are absent near the center of the response curve, but deep nulls appear at the blind speed near the two ends of the response.

The results obtained from this case and from the case when M=I is drawn in the fig. (C-1), for two and three period staggered to distinguish and analyse the optimum ratio, it has been noticed that the best case is the three periods staggered when M=N-I because when N increased more than 20, the deepest minimum in the response curve will be constant.

It has been noticed that if the ratio of periods is very close say 29:30 then the blind speed is increased by a large factor, in this case 30 times. On the other hand the two components first –order blind speeds are separated by only one part in 30 so that the response falls nearly to zero after the first maximum. The choice of stagger ratio is clearly a compromise between gap free coverage at high target speeds and deep minimum at low speeds. The final selection had been made by examining computed responses for various ratios.

As the stagger ratio T1 / T2 is increased, the depth of the nulls between blind speeds in the response characteristics will increase too, and when this ratio is equal (1 / 3, 1 / 4, 3 / 4, …etc.) more than one deepest minimum can be obtained in the response curve each equal (-50dB) and it is considered to be unsatisfactory case, but the worst case is when the staggered ratio is 1/2 and the last result can be achieved when:

- N=10 , M=5
- N=20 , M= 10
- N=30, M= 15

It was also found that if the value of N is increased, the value of the improvement factor will be decreased, i. e. we must choose the value of N as small as possible.

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